The binomial probability applies to experiments for which the following is true:

- a) the experiment consists of repeated trials
- b) each trial is independent
- c) each trial has only two possible outcomes

Situations in the real world which have binomial probabilities are flipping a coin (head or tail), hitting a bulls-eye (hit or miss), making a foul shot (yes or no), and birth of a child.

The binomial probability function is a handy formula to calculate probabilities in binomial situations.

\[ P(x) = \binom{n}{x} p^x q^{n-x} \]

Let's break this down:

\[ P(\text{number of successes}) = \binom{n}{x} \text{ prob of } p \begin{matrix} x \end{matrix} \text{ successes} \binom{p}{p} \text{ prob of } q \begin{matrix} n-x \end{matrix} \text{ failures} \]

Example 1
A balanced coin is tossed 4 times. What is the probability of getting 3 heads?

- \( n = 4 \) tosses
- \( x = \) successes, i.e., heads = 3
- \( p = 0.5 \), probability of getting a head in one toss

\[
P(3 \text{ heads in 4 tosses}) = \binom{4}{3} (0.5)^3 (0.5)^1 = 4 \times 0.125 = 0.5 = 25\%
\]

Example 2
5 cards are drawn from a deck, one at a time and then replaced. What is the probability that at least 3 of them will be red?

First, let's determine if this is a binomial situation, using a) through c) from above.

- a) repeated trials? Yes. 1 card at a time, 5 times
- b) independent trials? Yes. Replacing the cards after they are drawn keeps each trial unaffected by previous outcomes.
- c) only two outcomes? Yes. Red or not-red (black)

It is binomial, so we can use the formula.

\[
p_5(\text{red in 5 draws}) = \frac{5}{5} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 0.3125
\]

\[
p_4(\text{red in 5 draws}) = \frac{5}{5} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 0.15625
\]

\[
p_5(\text{red in 5 draws}) = \frac{5}{5} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 0.03125
\]

The total probability is the sum of these, or

** The probability of each situation which is a success must be calculated separately then added together.