QUICKHELP

Differential Calculus

VOCABULARY & NOTATION

The derivative of a function \( f(x) \) can be written as:

\[ f(x) \text{ or } \frac{df}{dx}(x) \text{ or } [f(x)]' \]

The derivative tells you:
- the slope of the tangent line at a point
- the instantaneous rate of change at a point

Function notation for \( f(x) \) and \( g(x) \) is shortened to only \( f \) and \( g \) occasionally for clarity.

These first 3 rules are important! All the remaining rules contain these.

Power Rule
\[
\left(x^n\right)' = nx^{n-1} \quad \text{when } x \neq 0
\]
When \( x \) is raised to a constant \((\neq 0)\), multiply the original function by the exponent and subtract one from the exponent.

ex. \( (x^3)' = 3x^2 \)

Constant times a function
\[
\left(c \cdot f(x)\right)' = c \cdot f'(x)
\]
Find the derivative of the variable term as you normally would (use power rule). Multiply the result by the constant.

ex. \( (\pi \cdot x^4)' = \pi \cdot (x^4)' = \pi \cdot 4x^3 \)

Derivative of a Constant
For any number, \( c \)
\[
f(x) = c \Rightarrow f'(x) = 0
\]
The derivative of a constant is always 0.

\[
f'(x) \text{ when } x = 13 \Rightarrow f'(13) = 0
\]

The following shortcuts contain the rules above combined in different ways.

Sums and Differences
\[
\left[f + g\right]' = f' + g' \quad \text{and} \quad \left[f - g\right]' = f' - g'
\]
The derivative of a sum (or difference) is the sum (or difference) of the derivatives of each part, found separately.

ex. \( (x^4 - 2x^3)' = (x^4)' - (2x^3)' = 4x^3 - 6x^2 \)

Product Rule
\[
\left[f \cdot g\right]' = f' \cdot g + f \cdot g'
\]
The derivative of a product = the first factor \((f)\) times the derivative of the second factor \((g)\) + the second factor \((g)\) times the derivative of the first \((f)\)

ex. \( (x^2)(x - 1)' = (x^2)'(x - 1) + x^2(x - 1)' = x^2 + 2x(x - 1) = 3x^2 - 2x \)

Quotient Rule

\[
\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}
\]
The derivative of a ratio = [(derivative of numerator times denominator) - (numerator times derivative of denominator)] all divided by (denominator)²

ex. \( \left(\frac{x^2}{x - 1}\right)' = \frac{(2x)(x - 1) - x^2(1)}{(x - 1)^2} = \frac{2x^2 - 2x - x^2}{x^2 - 2x + 1} = \frac{x^2 - 2x}{x^2 - 2x + 1} \)

Techniques of Differentiation

Intro to Techniques of Differentiation
aka: the “shortcuts”

Instead of using the general purpose formula:

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]
to find the equation for a derivative, there are shortcuts used for various types of equations.

Trigonometric Functions

Think of the derivatives of trig functions as definitions: no work involved, they just are.

\[
\begin{align*}
\sin' x &= \cos x \\
\cos' x &= -\sin x \\
\tan' x &= \sec^2 x \\
\csc' x &= -\cot x \\
\sec' x &= \sec x \tan x \\
\cot' x &= -\csc^2 x
\end{align*}
\]