QUICKHELP

MAT201 Differential Calculus

The Chain Rule

VOCABULARY & NOTATION

Composite Function - one function inside another function.

 $f \circ g$ is the same as $f(g(\mathbf{x}))$

Exam

The chain rule means: $\left[f(g(\mathbf{x}))\right] = f'(g(\mathbf{x}) \cdot g'(\mathbf{x}))$.

To simplify this notation, substitute u for g(x). The original equation is now rewritten as $\int f(u) = f'(u) u'$. Multiply the derivatives of f and of u together to determine the derivative of the original composite function.

ple 1: Find the derivative for:
$$y = (x^2 - x + 1)^{23}$$

Instead of multiplying $x^2 - x + 1$ by itself 23 times and *then* taking the derivative, we can use the chain rule.

First, pick the equation for the u-substitution and find its derivative.

> Let $u = x^2 - x + 1$ So: u' = 2x - 1

Keep u' on the side for now. Rewrite the original equation with u.

The *new* original equation is now $y = u^{23}$. This is easily differentiated through the power rule: $y' = 23u^{22}$

The chain rule says this derivative is multiplied by u'. DO NOT FORGET THE u' ON THE END: $y' = 23u^{22} \cdot u'$

The Chain Rule allows composite functions to be easily differentiated by breaking them into smaller parts.

The derivative of each piece will be found separately and then

Example 2: Find the derivative for: $y = sin(x^2)$

Choose u and differentiate it

combined together.

$$u = x^2 \qquad \qquad u' = 2x$$

The original equation is now written as: y = sin(u)

Differentiate this using the rule for sin y' = cos(u) u'

Substitute u into the equation for the answer

What do I pick for u? u should be chosen so that the derivative can be found easily. If the result is not easily differentiable, choose something else for u.

You can run the chain rule on u again as many times as need be. The following example will show you the idea. $y = sin\left(\left(x^2 - 1\right)^2\right) \rightarrow If u = \left(x^2 - 1\right)^2$ then y = sin(u) and y' = sin(u) u' BUT u' requires another chain rule! So let $u = v^2$ and $v = x^2 - 1$ That gives us $\rightarrow u' = 2v \cdot v$ and v = 2x Just substitute back into the y = equation one variable at a time $\rightarrow y = sin(v^2) \cdot (v \cdot v') \rightarrow y' = sin(x^2 - 1)^2 \cdot (2(x^2 - 1) \cdot 2x)$ (this can be simplified with a little algebra)

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